On the measurement of gyroscope spin axis using the London Moment of a rotating superconductor

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Abstract. The aim of this essay is to discuss and approach the problem of determining the spin axis direction of high precision gyroscopes. A solution is presented which uses the London Moment of a rotating superconductor. The Gravity Probe B gyroscopes are consistently referred to as a supporting case study. Justification for the use of these methods is given when such high accuracy is demanded.

1. Introduction

Measuring the axial orientation of a near perfect gyroscope to a high accuracy, whilst not disturbing its motion is a complex problem in physics and engineering. The key question, "How do you measure a spinning, perfectly uniform sphere that has no marks on it?" was asked by Francis Everitt of Stanford University, the principal investigator of NASA's Gravity Probe B (GP-B) project [1]; Everitt was referring to the problem of determining the spin axis of the most perfect gyroscopes ever made, which were used by the GP-B team to measure miniscule effects predicted by Einstein's theory of General Relativity. The requirements placed upon the gyroscopes used in the GP-B project are discussed. The interesting and surprising effects of superconductivity present a solution to the problem. The use of superconductivity is furthermore discussed and justified by its use in the GP-B investigation.

“Gyroscope” is a general term for any device that consists of a mass rotating about an axis. The name was given in the 19th century when Foucault, a French scientist devised an experiment to demonstrate the rotation of the Earth. This involved a rotating wheel to provide a fixed reference frame to which the rotation of the Earth could be compared [2]. A conventional gyroscope is a symmetric rigid body undergoing rotation about a symmetry axis, which, by some mechanism is given freedom of movement about axes perpendicular to its direction of spin [3]. A common method for obtaining such freedom is a gimbal, a set of pivoted frames to which the gyroscope is mounted [4]. At this point it should be noted that throughout the rest of the text a conventional gyroscope will be referred to as a gyroscope.

The main errors in devices which use gyroscopes come from unwanted friction and external torques. The attempt to obtain a gyroscope completely free of friction and torque has been a great engineering problem for centuries. Many novel devices have been invented to reduce these effects as much as possible [2]. Historically, measurements of the spin axis of a gyroscope would be by mechanical means and this would cause a reaction force significant enough to affect the motion of the gyroscope.

2. The Physics of Gyroscopes

The physics of a gyroscope is described entirely by classical mechanics. The primary equation describing the motion of a gyroscope is Newton’s Second Law of motion for rotating bodies.

$$\frac{d\underline{L}}{dt} = \sum \underline{\tau}_{ext} \tag{1}$$

The equation of motion, (1) states that the rate of change of the angular momentum vector is equal to the sum of external torques acting on the body. If no net external torque acts then the angular momentum vector does not change with time and is said to be conserved [5]. The motional characteristics of a gyroscope can be derived from (1).

Gyroscopic inertia describes the effect whereby the axle of a rotating gyroscope tends to remain in the same orientation provided that the gyroscope is free of net external torque. This can be seen from (1) since in the absence of torque, the magnitude and direction of the angular momentum cannot change with time. When an external torque is applied to a spinning gyroscope, e.g. a spinning top in the Earth's gravitational field the axis direction does not move in the direction of the force but at right angles to it. The axis thus draws out an inverted cone in space; this motion is known as gyroscopic precession [2]. There exists another possible motion called nutation, in which the axis of rotation wobbles whilst it precesses. See [3].

The property of gyroscopic inertia has many applications in engineering including its usefulness as a stabiliser or vibration absorber in ships. A huge gyroscope provides resistance to the rolling torques created by waves [3]. The axle of a gyroscope remains fixed in space; this makes the gyroscope useful for navigation, whilst not having to rely upon external references such as the stars. Inertial navigation systems use a combination of gyroscopes and accelerometers to determine the position in 3D space of aircraft and missiles [3].

Gyroscopes are useful in many modern applications, such as satellites which use gyroscopes to stabilise their orbit. A famous example of this is the Hubble Space Telescope. Hubble is able to point accurately at celestial objects by using 6 gyroscopes to stabilize itself. Due to the failure of 4 gyroscopes, Hubble is currently running on 2; this makes pointing to specific areas in the sky difficult, if another gyroscope fails, Hubble will become useless [6].

3. The Gravity Probe B Project

The GP-B probe, a joint project between NASA and Stanford University is one of the most recent and accurate tests for Einstein's theory of General Relativity. The project aims to test two effects predicted by General Relativity, namely the geodetic effect and the Lense-Thirring frame-dragging effect [7,8]. The geodetic effect is the amount by which a celestial body, such as the Earth curves the space-time around it. The Lense-Thirring frame dragging effect describes how a massive rotating body, such as the Earth drags its local space-time around with it

[9]. These independent effects were predicted by Leonard Schiff in 1960 to change the direction of the spin axis of a free gyroscope as it orbits the Earth [10]. The GP-B experiment involved measuring the change in spin axis direction of 4 gyroscopes, measured against a guide star [11]. A satellite was placed in a polar orbit; this meant that the two angular displacements caused by the predicted effects were orthogonal and hence easy to separate. The effects were calculated and shown to be miniscule; a shift in the spin axis in the plane of the satellites orbit of 1.8×10^{-3} degrees per year for the geodetic effect and a shift perpendicular to the plane of the orbit of 1.1×10^{-5} degrees per year for the frame dragging effect [12]. This investigation is still in the final stages of data analysis and results are expected by December 2007 [13].

Many new and unique technologies were created for the mission, the most interesting of which were those needed to create gyroscopes so perfect and free of torque that they could act as a local frame of reference. This frame of reference would then be used as a comparison to that of the guide star [12]. The gyroscopes were designed so that an accuracy of 1.4×10^{-7} degrees per year could be achieved in the measurement of the spin axis direction [9].

The gyroscope rotors were fused quartz spheres which were suspended electrically and cooled to a cryogenic temperature of 2K. The gyroscope rotors are nearly perfectly spherical with a radius of 1.9cm. They are smooth to 20 nanometres, only several atomic layers. They have density uniform to 3 parts in 10^7 [14]. Due to the method of suspension of the gyroscope rotors, the spheres must also be electrically spherical so that they possess a near zero electric dipole moment [11]. These are the most perfect spheres ever made by mankind and the only known object in the entire universe that is more spherical is a neutron star [15].

Here lies the problem; how do you determine the direction in which a perfect sphere is rotating when it has no marks on it, whilst at the same time not disturbing its motion significantly? The solution to this problem lies in the realms of low temperature physics, particularly the exotic state of matter known as superconductivity.

4. Superconductivity

Superconductivity was discovered by Kammerlingh Onnes in 1908, after he successfully liquefied Helium. After this remarkable triumph he used liquid Helium to measure how the electrical resistance of mercury changes at temperatures approaching absolute zero [16]. In 1911 he found that the metal’s resistivity dropped as expected until at about 4K at which point the resistivity discontinuously dropped to zero. He realised that below this temperature mercury had undergone a phase transition into a new state of matter with completely different electrical properties [17]. Since then many other metals, alloys, and other compounds have been shown to have a superconducting state, each characterised by a critical temperature of superconduction (T_c). An interesting property of a superconductor is that it has no electrical resistance, it is a perfect conductor. Strictly, this is only true for a direct current since there is some power dissipation in an alternating current circuit due to the changing electric field [18].

Superconductors have several magnetic properties of interest. The critical temperature of superconductivity decreases as the strength of an applied magnetic field is increased. If the applied magnetic field is strong enough it will prevent the transition into the superconducting state [18]. A well known magnetic property of a superconductor is the Meissner Effect. Here, a superconductor in a magnetic field no stronger than the critical value is cooled into the superconducting state. As the temperature drops below the critical temperature all magnetic flux lines are expelled from its interior leaving only surface (or screening) currents. This property is known as diamagnetism [19] and a superconductor is perfectly diamagnetic [20]. This effect shows that there is more to a superconductor than it just being a perfect conductor. The state of magnetisation of a perfect conductor depends on the order of events, in which cooling to the point of zero electrical resistance and the application of an applied magnetic field are performed. A perfect conductor may or may not contain a magnetic field in its interior depending on the sequence of events in which it arrived at its final state. However, a superconductor will always expel magnetic flux from its interior [18]. The perfect diamagnetism of a superconductor means that a permanent magnet placed above a superconductor will levitate. Since the rotation of the permanent magnet can be finely controlled it has been suggested that this effect could be used as a specimen manipulator [21].

Fritz London in 1908, proposed a phenomenological theory known as London Theory [18, 22] which attempts to explain the macroscopic properties of superconductors. The theory is classical and makes the assumption that not only the gradient of the magnetic field but the magnetic field itself decays rapidly in the interior of a superconductor. The theory predicts that the magnetic field decays exponentially inside a superconductor. The London penetration depth (λ_L) is defined as the distance from the surface at which the magnetic field strength falls by 1/e of its value at the surface. Although experimentally measured penetration depths are larger than predicted, the London Theory still makes good predictions about how a superconductor behaves [18]. A magnetic field in a superconductor satisfies

$$\nabla^2 \underline{B} = \frac{1}{\lambda_L^2} \underline{B} \tag{2}$$

The solutions to (2) show that a stationary superconductor will expel all magnetic flux from its interior except for near the surface. However, a rotating superconductor will generate a magnetic field. In this case there exists a magnetic field inside the bulk and no magnetic field near the surface. The magnetic field produced by a rotating superconductor is known as the London Moment [23,24,25,26] and has a magnitude given by

$$\underline{B} = -\frac{2m}{e} \underline{\omega} \tag{3}$$

Here, m and $-e$ are the mass and charge of the superelectrons respectively (the current carriers in the superconducting state) and ω is the spin rate of the superconductor. An explanation of the origin of the magnetic field comes from Becker *et al*, [27] who state that the field is produced by the superelectrons lagging behind the ion lattice at the surface [23]. During rotation an electric field is also induced, although this is not so well understood [28]. It is interesting to note that (3) provides a simple method of determining the charge-to-mass ratio of the electron [25].

5. The Perfect Solution

The solution to the problem of measuring the spin axis of a perfect sphere lies in (3). Remarkably, the direction of the London Moment is exactly aligned with the spin axis direction. The London moment is an extremely small magnetic field (of the order of 10^{-8} Tesla for a superconductor spinning at 170Hz [26]) and so a highly sensitive magnetometer is required. A magnetometer is any device which can be used to measure the size and direction of a magnetic field [29]. One of the most sensitive magnetometers currently available is known as a SQUID (Superconducting Quantum Interference Device) [30]. SQUIDs also exploit the properties of superconducting materials. To understand how a SQUID works a microscopic description of superconductivity is required.

A popular theory is the BCS theory [31], in which pairs of electrons experience a weak attraction and join into bound states known as Cooper Pairs by the transfer of phonons [32]. A phonon is quantised vibrational energy. The principle operation of a SQUID relies upon the Josephson Effect, [20] in which Cooper Pairs can tunnel from one superconducting material, across a thin insulator to another superconducting material. The arrangement of two superconductors separated by a thin insulator is known as a Josephson Junction. See [33] for superconductive tunnelling and its potential uses, such as in SQUID devices.

GP-B scientists realised that if they coated their spherical gyroscope rotor with a superconducting material then the London moment induced as the rotors spun would exactly align with the spin axis. This is how the axis of rotation of a perfect sphere with no marks on it is measured without disturbing its motion. The GP-B gyroscope rotors were coated in a layer of niobium 1.25 micrometers thick [12]. Niobium begins to superconduct at 9.46K [17]. The SQUID detected changes in magnetic flux and was able to measure the change in spin direction to high precision, well exceeding the requirements for detecting the geodetic and frame dragging effects [34].

6. Conclusion

In conclusion, this essay has discussed the problem of measuring the spin axis direction of a near perfect gyroscope. The London moment provides a clever solution since the magnetic field direction is exactly aligned with the spin axis; these measurements do not significantly disturb the motion of the gyroscope.

The use of high precision spherical gyroscope rotors that were free of torque was an important requirement for the Gravity Probe B team. The gyroscopes were used to measure miniscule changes in the curvature of space-time predicted by Einstein’s theory of General Relativity.

The release of the final results of the GP-B experiment has been delayed for 8 months due to unexpected torque on the gyroscopes [13]. Extra work has been needed to model out these effects. This shows the difficulty of making a completely free gyroscope when effects so small must be considered. However, it is impossible to have a perfectly free gyroscope and make measurements on it without disturbing its motion in some way.

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